ENGR 326
ODE - RKF Lab Assignment

As part of a new waste discharge permit, a community is facing a requirement that the concentration of phosphorus in their wastewater treatment plant effluent be reduced. Currently, the concentration in the discharge results in a phosphorus concentration in the stream receiving water of 0.85 mg/l. The new phosphorus discharge limit is to be determined based on a requirement that the phosphorus load from the effluent result in a concentration less than 0.05 mg/l at a point 43.2 km downstream from the point of discharge. The phosphorus limit at the downstream point is set to protect beneficial uses of the stream for recreation and domestic water supply.

The engineer in charge of the treatment plant upgrade has determined phosphorus in the stream is removed by sedimentation and algal uptake. She has also determined that phosphorus is the limiting nutrient for algal growth in the stream, and that the algal growth rate is adequately described by Michaelis-Menton saturation kinetics. After literature and laboratory investigations, the engineer determines that the following equations describe the time rate of change in phosphorus and chlorophyll-A (algae cells) in the stream.

$$\frac{dP}{dt} = -K_{P_1}P - K_{P_2}\mu A$$
$$\frac{dA}{dt} = -K_{A_1}A + \mu A$$

where

P = phosphorus concentration (mg/l)
A = chlorophyll-A concentration (mg/l)
t = time (days)
K_{P_1} = first order removal rate of phosphorus by settling (1/day)
K_{P_2} = yield coefficient (mg phosphorus/mg chlorophyll-A)
K_{P_3} = half saturation concentration for phosphorus (mg/l)
K_{A_1} = algal death rate (1/day)
μ = algal growth rate (1/day)

$$= \mu\text{max}\left(\frac{P}{K_{P_3} + P}\right)$$

μmax = maximum algal growth rate (1/day)

Write a program which determines the concentration of phosphorus at the downstream surveillance point as a function of the concentration in the effluent by using the Runge Kutta Fehlberg method to solve the differential equations above. Use the program to determine the new maximum concentration of phosphorus in the effluent of the treatment facility that will result is meeting the downstream phosphorus limit. Assume that the average stream velocity is 0.05 m/sec, $K_{P_1} = 0.05$/day, $K_{P_2} = 1.0$, $K_{P_3} = 0.025$ mg/l, $K_{A_1} = 0.003$/day, $\mu\text{max} = 0.42$/day, and the upstream concentration of chlorophyll-A in the stream is 0.002 mg/l. Provide a plot of the concentration of phosphorus from the point of discharge to the downstream monitoring point.