Romberg Integration Algorithm

Solve

$$T = \int_a^b f(x)dx$$

using the trapezoid rule as the base integration technique, beginning with nt trapezoids. Note that the trapezoid rule tuncation error has the form $c_1h^2+c_2h^4+c_3h^6+\cdots$ (Atkinson, 1989, *Introduction to Numerical analysis, second edition*, page 266).

$r = 2$! ratio of long to short stepsize
$h = (b-a)/nt$! stepsize = (upper limit - lower limit)/number of trapezoids
$s_1 = [f(a) + f(b)]/2$
$s_2 = \sum_{i=1}^{nt-1} f(a + i \cdot h)$
$T(1,1) = h(s_1 + s_2)$
$n = 1$! number of iterations
do
  $nt = 2 \cdot nt$! double the number of trapezoids
  $h = (b-a)/nt$! calculate the new stepsize
  $s_2 = s_2 + \sum_{i=1}^{nt/2} f[a + h(2i-1)]$! update s_2
  $T(n+1,1) = h(s_1 + s_2)$
  do $j = 2, n+1$! Do the extrapolations
    $T(n+1,j) = \frac{r^{2(j-1)}T(n+1,j-1) - T(n,j-1)}{r^{2(j-1)}-1}$
  end do
  if stopping criteria met - exit
  $n = n + 1$
end do

Reasonable stopping criteria

1. Algorithm converged $\left(\left| \frac{T(n+1,n+1) - T(n+1,n)}{T(n+1,n+1)} \right| < \epsilon \right)$
2. Iteration limit ($n >$ max iterations)